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W H E N W O R K E R S S A V E A N D I N V E S T :

S O M E K A L D O R I A N D Y N A M I C S

by

K U M A R A S W A M Y V E L U P I L L A I

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1. Introduction

In advanced capitalist societies it is clear that workers' standard of living, in the aggregate, is well above the so-called subsistence minimum. In this situation, along Kaldorian lines, we may investigate the possibilities and consequences of allowing savings out of wages in addition to savings out of profits. Now, if workers save or if savings are made, in the aggregate, by representatives of the class, out of wages (and contractual income in general) then they will own a part of the economy's stock of capital. If they own a part of this stock of capital, they should, naturally, be allowed to see it develop, subject to constraints, according to their own criteria. A capitalist could be assumed to invest according to various types of profit criteria. These criteria need not, and will not, motivate workers. It is perhaps more realistic to assume that workers strive to achieve full employment at the cost of most everything else. It was a similar sort of reasoning that made Pasinetti question Kaldor's original theory (cf. Pasinetti (1974) Ch. V and Kaldor (1955-6)):

There is a logical slip, in the (Kaldorian) theory . . . , which has so far passed unnoticed. The authors have neglected the important fact that, in any type of society, when any individual saves a part of his income, he must also be allowed to own it, otherwise he would not save at all. . . . And since ownership of capital entitles the owner to a rate of interest, if workers have saved--and thus own a part of the stock of capital . . . --then they will also receive a share of the total profits. Therefore total profits themselves must be divided into two categories: profits which accrue to the capitalist and profits which accrue to the workers.

(Pasinetti (1974), pp. 106-107)

We may then wonder whether there is not also a 'logical slip' in Pasinetti's extension, if workers own a part of the total stock of capital, then not only should they receive the ruling rate of interest on that part of the capital stock which they own--but also should be allowed to influence the direction of future developments in the scale and composition of the capital stock. In other words, should we not modify the concept of a single investment equation in a macrodynamic model? It is the consequences, for the level of employment and functional shares, of the removal of this simple assumption--that workers do not influence investment decisions--that we investigate in this paper.

It may be recalled that in the Kaldor-Pasinetti approach to the so-called Post-Keynesian or Neo-Kaldorian theory of functional income distribution there is a clear distinction between different propensities to save out of different types of income (contractual income vs. income from property) as against different propensities from different classes of income receivers (workers vs. capitalists). Kaldor, in particular, has stressed the former distinction whereas Pasinetti worked with the latter distinction in his celebrated paper. We take the Pasinetti approach and work with the distinction between different classes of income receivers--i.e., workers and capitalists.

Further, in these early and justly famous papers the savings propensities were assumed to be constant. This is quite obviously a very unrealistic assumption; however, the most damaging assumptions are related to full employment and the long-run equilibrium framework.

Using simple but not unrealistic assumptions about typical macroeconomic variables we construct a model to analyze

the dynamical interaction between wages, profits and employment when both workers and capitalists invest--albeit for achieving different goals.

2. The Model

We assume a closed, essentially noncompetitive one good economy with no explicit role for government. These assumptions can be easily relaxed. We postulate behavioural equations for five crucial variables: money-wages, prices, productivity, capacity utilization (and for labour hoarding) and investment, to derive from these a dynamical system of general non-linear differential equations in the share of wages and the employment ratio.

(a) Money-wage dynamics:

$$\frac{\dot{m}}{m} = f \left[v, \frac{\dot{p}}{p}, \left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} \right) \right] \quad (1)$$

and $f' > 0$ w.r.t. all arguments

$f \in C^1$, for $m \geq \bar{m}$.

where: m : money-wages

$v = L/N$: employment ratio

L : employed labour force

N : available labour force (labour supply)

p : prices

Y : output level.

Thus, according to equation (1), the dynamics of the proportional growth in money-wages are determined by workers' bargaining power (v), inflation (\dot{p}/p) and productivity $\left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} \right)$.

(b) Price formation:

$$\frac{\dot{p}}{p} = \lambda (\log m - \log p + \log \pi - \log Y + \log L) \quad (2)$$

$$\text{i.e., } \frac{\dot{p}}{p} = \lambda (\log \pi - \log u) \quad (3)$$

where: $u = \frac{mL}{pY}$: share of wages

$\pi > 1$ (mark-up factor)

$\lambda > 0$ (adjustment coefficient)

Rewriting (3) in a more general way, we have:

$$\frac{\dot{p}}{p} = g(u; \lambda, \pi) \quad (4)$$

where $g_u' > 0$ and $g \in C^1$.

The simple hypothesis underlying the above price equation is that actual prices adjust to equilibrium prices with a simple exponentially distributed log--the value of the adjustment coefficient and the mark-up factor are proxies for capitalists' power to counteract adverse money-wage dynamics. It is assumed, of course, that prices are set by capitalists.

(c) Productivity:

$$\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = h \left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L} \right) \quad (5)$$

and $h \in C^1$.

where: $K =$ total capacity (capital stock) in the economy. Equation (5) encapsulates the classic Kaldorian technical progress function in almost its full non-linear generality.

(d) Capacity Utilization (and Labour Hoarding):

We will assume that the desired utilization level of capacity is achieved only when employment is full, i.e., when $v = 1$. One way of representing this relationship (and at the

same time enabling us to obtain long-run constancy in the capital-output ratio with short-run labour-hoarding) is to posit the following:

$$k = \frac{K}{Y} = q(v) \quad (6)$$

$$\text{where } q' < 0 \text{ and } q \in C^1 \\ q'' > 0$$

(e) Investment Equations:

We now come to the crucial relationship: the investment behaviour by capitalists and workers.

$$K = K_C + K_W \quad (7)$$

where: K_C : capital stock owned by capitalists

K_W : capital stock owned by workers.

Taking slightly modified version(s) of the first of Kaldor's three investment equations which he presented in his now famous growth model of 1957 (the second and third versions were developed in the 'Corfu Model' and the 'Kaldor-Mirrlees' model; cf. Kaldor (1957), (1961), and Kaldor-Mirrlees (1962)), we get:

$$\dot{K}_W = K_W \frac{\dot{Y}}{Y} + s_W(v, u) u \cdot Y \quad (8)$$

$$\dot{K}_C = K_C \frac{\dot{Y}}{Y} + s_C(u) \cdot (1 - u) Y \quad (9)$$

The first term in either equation reflects the assumption that the respective classes increase the stock of capital owned by them in proportion to the proportional growth in output as a whole--i.e., to maintain 'own' capital-output ratios, to be viewed as a variant of the accelerator principle. The second term, on the other hand, shows the ex-ante savings intentions of each of the classes. It could be expected that

if unemployment was increasing, $s_w(v, u)$ would be high or if the share of wages were too high, $s_c(u)$ would be low. The former emphasizing the employment criterion, the latter the profit criterion. Unlike the restrictive Kaldor-Pasinetti framework, where savings propensities were constant, we are able not only to introduce an element of explicit conflict, but also, for the system as a whole, an element of complementarity. From the above, the aggregate economy-wide investment equation would be:

$$\frac{\dot{K}_w + \dot{K}_c}{Y} = \frac{\dot{K}}{Y} = \frac{\dot{Y}}{Y} \cdot \frac{1}{Y} (K_w + K_c) + s_w(v, u) \cdot u + s_c(u) (1 - u) \quad (10)$$

Substituting (6) in (10) and rearranging we get:

$$\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} + s_w(v, u) \cdot u \cdot \frac{1}{q(v)} + s_c(u) \cdot (1 - u) \frac{1}{q(v)} \quad (11)$$

Now, equations (1) - (11) constitute the basic elements of the model. We proceed to derive the reduced-form equations for the employment-ratio and the share of wages as follows. From (6) we get

$$\frac{\dot{K}}{K} - \frac{\dot{Y}}{Y} = \frac{q'(v)}{q(v)} \cdot \dot{v} \quad (12)$$

Using (11) and (12) we get:

$$\frac{\dot{v}}{v} = \frac{1}{q'(v)} \cdot \frac{1}{v} \{s_w(v, u) \cdot u + s_c(u) \cdot (1 - u)\} \quad (13)$$

$$\text{or} \quad \frac{\dot{v}}{v} = G(v, u) \quad (14)$$

$$\text{where} \quad G(v, u) = \frac{1}{q'(v)} \cdot \frac{1}{v} \{s_w(v, u) \cdot u + s_c(u) (1 - u)\} \quad (15)$$

From (5) and (11) we get:

$$\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = h \left\{ \left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} \right) + \left[s_w(u, v) \cdot u \cdot \frac{1}{q(v)} + s_c(u) \cdot (1 - u) \frac{1}{q(v)} \right] \right\} \quad (16)$$

Assuming separability, we have:

$$\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = h_1 \left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} \right) + h_2 \left[s_w(u,v) \cdot u \cdot \frac{1}{q(v)} + s_c(u)(1-u) \frac{1}{q(v)} \right] \quad (17)$$

$$\text{Therefore: } B \left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} \right) = h_2 [\quad] \quad (18)$$

$$\text{Thus: } \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = B^{-1} \{ h_2 [\quad] \} = D \{ h_2 [\quad] \} = D(u,v) . \quad (19)$$

Then, finally, from $u = \frac{mL}{pY}$ we get:

$$\frac{\dot{u}}{u} = \frac{\dot{m}}{m} - \frac{\dot{p}}{p} - \left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} \right) \quad (20)$$

Substituting (1), (4) and (19) appropriately we get:

$$\frac{\dot{u}}{u} = H(u,v) \quad (21)$$

where:

$$H(u,v) = f[v, g(u; \lambda, \pi), D(u,v)] - g(u; \lambda, \pi) - D(u,v) \quad (22)$$

It is easily seen that long-run constancy in the share of wages can be achieved if money-wage dynamics are such that wage rises are due to perfect anticipation (expectations) of inflation (absence of money illusion) and growth in productivity. Clearly, inflation can play a decisive role in re-distributing shares, whereas productivity seems to be the determining factor in the secular rise in real wage rates. The crucial reduced-form dynamical system in the employment ratio and the share of wages is given by (14) and (21) taken simultaneously. We now analyze the (dis-equilibrium) dynamical properties of this system under some standard assumptions about the functional forms.

3. Dynamics of the Model

In the standard Kaldor-Pasinetti framework, two important relations have to be satisfied. The first is the inequality relation between the two savings propensities; the second is Pasinetti's assumption about the amount of savings made by workers who receive profits. More precisely:

. . . in the long run, when workers save, they receive an amount of profits . . . such as to make their total savings exactly equal to the amount that the capitalists would have saved out of workers' profits . . . if these profits remained to them.

(Pasinetti, op. cit., p. 111)

The assumptions we shall make to generate equilibrium dynamics from any disequilibrium situation (in the phase-plane of u and v) will be comparable to the above two relations. However, we shall not be able to rule out possible long-run underemployment equilibria--indeed, we do not wish to do so, since one of the main purposes of the analysis presented here is to try to show what workers can do to maintain full employment. Implicitly, we also argue for a reorientation of workers' strategies in wage bargaining--if full employment (and share in output) is (are) the dominant target(s).

Clearly, we have, first of all, to investigate the signs of the following four partials:

$$a) \frac{\partial G}{\partial u}, \quad b) \frac{\partial H}{\partial v}, \quad c) \frac{\partial G}{\partial v} \quad \text{and} \quad d) \frac{\partial H}{\partial u}$$

if we are to make any meaningful propositions about qualitative dynamics. Now, direct computation gives us the following:

$$\frac{\partial G}{\partial u} = \frac{1}{q'(v)} \cdot \frac{1}{v} \{ [s_w(v, u) - s_c(u)] + \left[\frac{s_w(v, u)}{u} - \frac{s_c(u)}{u} \right] + \frac{s_c(u)}{u} \} \quad (23)$$

The influence of increasing share of wages on the unemployment ratio will be the resultant of two, partly, opposing forces. On the one hand capitalists' propensity to invest may be blunted; on the other hand, workers, to the extent that full employment is important, will have to compensate for any such negative impact on investment from the capitalists' side. This is precisely the assumption made by Pasinetti. In our case, therefore, this will be in terms of a relationship between $\frac{\partial s_w}{\partial u}$ and $\frac{\partial s_u}{\partial u}$. For simplicity let us assume that changes in these two factors exactly compensate each other. Then, the sign of (23) depends on the relationship between the absolute values of the savings propensities --since, by assumption $\frac{\partial s_c}{\partial u}$ and $q'(v)$ are negative. Now we make the Kaldorian assumption explicitly, i.e., that the savings propensities of capitalists are greater than those of the workers. Then the first term inside the curly brackets will be negative; the second term by assumption zero; the last term negative. Thus:

$$\frac{\partial G}{\partial u} > 0 \quad (24)$$

Again, direct computation gives:

$$\frac{\partial H}{\partial v} = \frac{\partial f}{\partial v} - \frac{\partial D}{\partial v} \quad (25)$$

In this case, it is quite reasonable, economically, to assume that the effects on money-wage rises of increasing employment are greater than those on productivity. (We shall comment on the opposite assumption at a later stage-- cf. below p. .) Then:

$$\frac{\partial H}{\partial v} > 0 \quad (26)$$

Next we have:

$$\frac{\partial H}{\partial u} = \frac{\partial f}{\partial u} - \frac{\partial \pi}{\partial u} - \frac{\partial D}{\partial u} \quad (27)$$

The redistributive role of inflation and thus, implicitly, the relative strengths of capital and labour is crucial in determining the sign of this relation. If we assume that capitalists' pricing policies are more aggressive than workers' bargaining capabilities then clearly $\frac{\partial H}{\partial u}$ will be negative. Put another way, this means that unless workers can more than compensate for inflation by perfect expectations, capitalists can choose the mark-up factor and the adjustment coefficient in such a way that they will at least (in the long run) preserve their share of output. If this is the case, then:

$$\frac{\partial H}{\partial u} < 0 \quad (28)$$

Finally, we have:

$$\frac{\partial G}{\partial v} = \frac{q'(v) \cdot v \cdot \frac{\partial s_w(v, u)}{\partial v} - [s_w(v, u) \cdot u + s_c(u) - u \cdot s_c(u)] \cdot [q''(v) \cdot v + q'(v)]}{[q'(v) \cdot v]^2} \quad (29)$$

By assumption $q'(v)$ and $q''(v)$ are negative and positive respectively. To be consistent with our earlier assumptions and discussions we have to assume that $\frac{\partial s_w}{\partial v}$ is positive. Since we have assumed $s_c > s_w$ and since also $u < 1$, it is clear that $\{s_c(u) - [(s_c(u) - s_w(v, u)) u]\}$ will be positive. The relationship between the absolute values of q'' and q' is more complicated. However, for any empirical form of $q(v)$ (example: $q(v) = k^* v^{-\mu}$, where k^* : desired capacity utilization; $0 < \mu < 1$) $|q''(v)|$ will be greater than $q'(v)$. This means that with standard labour hoarding assumptions, most of the adjustments are absorbed by changes in the rate of utili-

zation of capacity. Under these assumptions then:

$$\frac{\partial G}{\partial v} < 0 \quad (30)$$

Relations (23) - (30) and the underlying economic assumptions determine the general nature of the motion of the dynamical system in the phase-plane of the share of wages and the unemployment ratio. A typical possibility is shown in figure 1. We omit technical theorems about existence, uniqueness, stability (both global asymptotic and structural). By introducing, explicitly, the possibilities for workers to influence the rate of growth of output, productivity and capacity we are able to see how a point in the u - v plane such that $v = 1$ can be reached. To some extent we are, in this framework, able to vindicate Kaldor's intuition about monotone long-run tendencies towards stable 'more-or-less' full employment equilibria and constant shares (represented by B in fig. 1).

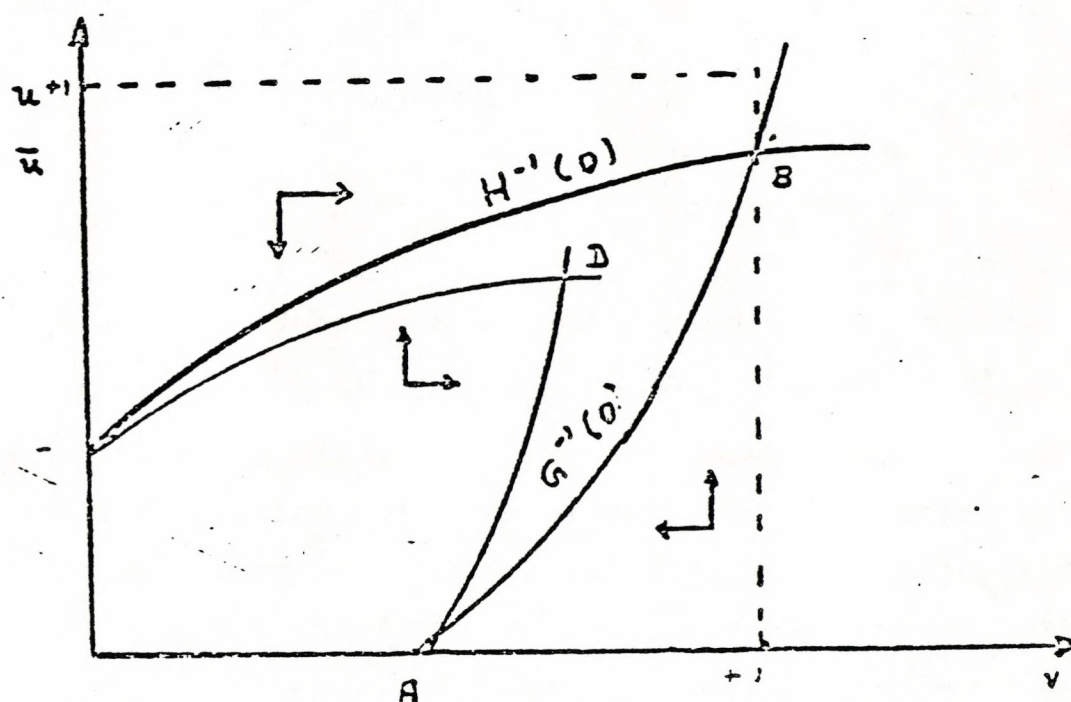


Figure 1.

It is clear, from inspection of figure 1, that any feasible equilibrium will be at least locally asymptotically stable. Under some very mild additional conditions global asymptotic stability and structural stability can be proved. It is also clear that any equilibrium will be unique under the conditions specified in this section.

Economically, existence of the capitalist class presupposes positive share to that group. This means u is strictly less than unity, say $\bar{u} \ll 1$. For any $u > \bar{u}$ capitalists could be expected to react by cutting down investment. The adverse effects on employment must then be counteracted by stimulating investment by workers. Thus, for example, from any disequilibrium position above \bar{u} the economic dynamics towards an equilibrium would be as indicated by the arrows in point I. The increased investment by the workers, to preserve or stimulate the level of employment, by increasing productivity via the technical progress function restores that balance in the share of output consistent with capitalists' desires. This latter is brought about by the interaction between money-wage rates rising more due to productivity effects (either by deliberate policy restraint by workers or by some other means) than inflationary or bargaining effects--and the possible flexibility in manipulating the adjustment coefficient and the mark-up factor in the dynamics of the price equation.

It must, therefore, be possible to imagine that most of the functional forms change as a function of the level and rate of economic activity. This, as we point out below in section 4, means that somewhat more sophisticated analysis is required for complete description.

Similar to the upper bound on u , as a necessary condi-

tion for the existence of the capitalist class, there will be a lower bound on v that will be acceptable to the workers. This, in addition to a lower bound on money-wage rates and constraints on the mark-up factor (political, institutional or otherwise) and the adjustment coefficient will effectively set limits in the positive orthant circumscribed by the origin and $u = 1$ and $v = 1$.

Whether there exist feasible equilibrium or not, and in the latter case how the functional forms change as the various limits (upper bound on $u = \bar{u}$, or natural limits $u = 1$, $v = 1$, etc.) are approached requires detailed empirical investigation which is beyond the scope of the limited aims of this paper.

4. Notes and Conclusions

So far, we have tried to remain within a Kaldorian framework (in terms of the technical progress function, investment function, inequalities w.r.t. the savings propensities, wage bargaining and mark-up pricing with an implicit assumption about labour hoarding). If we analyze the 'dual Pasinetti' case, inequalities w.r.t. savings propensities reversed, with more or less all other assumptions intact, the resulting dynamics in the u - v plane may generate limit cycles and, in the limit, a structurally unstable centre.

We have all along retained the two parameters related to the mark-up factor (π) and the adjustment coefficient (λ). The reason for this is that the most immediate generalization should be bifurcation analysis of the u - v system w.r.t. these two parameters. Obviously these two parameters are 'endogenous' and to that extent we can proceed along classical lines

in bifurcation analysis. However, if either of these parameters depends exclusively on time explicitly then (in particular, if the dual-Pasinetti case becomes important) the analysis should be in terms of that which is analogous to bifurcation analysis for non-autonomous systems (this is called 'branching of periodic solutions') (cf. Cronin (1980) Ch. 7, §A).

Three important directions in which the model should be generalized to take account of more realistic factors would be:

- a) Open economy
- b) Explicit introduction of the State
- and c) Introduction of Money in an essential way.

Qualitatively, (a) and (b) in simple dynamic models are not difficult to analyze within the framework of what has been presented above. However, the problem of introducing money or monetary factors in an essential way is more complicated. For example, an immediate generalization should be to incorporate a distinction between the (real) rate of profit and the money rate of interest in the investment equations and then expected inflation in a non-trivial way in the wage bargaining equation. This will require the phase-plane to be enlarged at least to a state-space of 3 dimensions. The problem, though not unmanageable, will involve, in analysis, loss of the two-dimensional elegance of geometry (and indeed the possibilities of invoking the powerful Poincaré-Bendixson theorem).

An even more important, and perhaps more interesting, direction in which to proceed would be to make more explicit the nature of the conflicting and complementary nature of the relationship between capital and labour. The most ele-

gant way, within the framework of dynamical systems, would be to use differential games. This means that the employment and shares criteria would be explicitly introduced in a criterion function for the workers and, say, profits criterion in that which represents capitalists' desires. Typically, this would be a non-zero sum game with possibilities for cooperative solutions. In particular, the introduction of the state (a third player in the game) and an open economy (rest of the world as a fourth player) fits naturally in a differential game framework.

In summary, then, we have been able to show, contrary to many popular interpretations of Kaldor (e.g. Brems (1979)) that within the framework of Kaldorian assumptions, when workers not only save, but also invest, full employment as a result, when it is a target, is not impossible--with long-run constancy in distributive shares. However s_w is no longer insignificant for the determination of the equilibrium configuration of the employment ratio and the share of wages.

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